INtegrating MainSTREAM Economic Indicators with Sustainable Development Objectives

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WP6: Costs of sustainability (general equilibrium analysis)

D 6.6b: Report on results of sensitivity analysis (ZEW)

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Non-technical Summary

The deliverable for WP6.6 describes the results of the sensitivity analysis for the climate policy analysis of WP6.1. Simulation exercises in economics as in other model based sciences depend on the choice of basic parameters of the model. While these themselves should be well founded on underlying assumptions only a thorough sensitivity analysis can establish the robustness of the deductions (or alternatively show weaknesses of the approach). In such an exercise the modeller analyses the measure of variation of key output variables of the model with respect to a sensible variation of input variables. In our case, we analyse the effect of a variation of Armington elasticities, which govern the relative value of imports and domestic products for a domestic customer, on economy-wide and sectoral competitiveness indicators. The results confirm the validity of the assertions of WP6.1 with exception of one indicator, the Relative Trade Balance (RTB) index, which reacts very sensitive to changes in the Armington elasticities. Across the robust indicators, there are also important differences: while the economy wide ToT are largely unaffected by the choice of the Armington elasticity, the magnitude of the sectoral indicators apparently depends on that choice. All results of the stochastic sensitivity analysis in this deliverable were obtained with the Gauss-Quadrature method proposed by Hermeling and Mennel (2008).

1. Introduction

"A fragile inference is not worth taking seriously. All scientific disciplines routinely subject their inferences to studies of fragility. Why should economics be different? ... What we need are organized sensitivity analyses."[1] The econometrician Edward Leamer emphasizes that economist using econometric as well as simulation methods should conduct regular sensitivity analysis to confirm the validity of their results. A sensitivity analysis is the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation in input parameters. It thus allows for an evaluation of the robustness of numerical results, as it translates the range (confidence intervals) of fundamental (input) parameters of the model into ranges (confidence intervals) of economic (output) variables.

Workpackage 6.6 commissions this important task for the simulation exercises in the INSTREAM project. In the following we will present the results of the sensitivity analysis of the competitiveness and climate policy study in Workpackage 6.1 (WP 6.1.) and the conclusions following from it. The presentation is organized as follows: In section 2 we briefly review the results of WP 6.1 to clarify the economic context, i.e. the impact of climate policy on the development of indicators of competitiveness in the CGE model PACE depicting the EU macroeconomy. Section 3 explains the methodology used in the sensitivity analysis of this deliverable, i.e. the Gauss-Quadrature approach developed in Hermeling and Mennel (2008) and its application to the current analysis. Section 4 presents the numerical results of the sensitivity analysis. Section 5 concludes.

2. Competitiveness and Climate Policy

The concept of competitiveness encompasses a number of factors that are not easily measurable in quantitative terms. But even though competitiveness is not precisely measurable indicators can approximate the level of competitiveness as a results-based notion concept and disclose the ability of a country, a sector or an industry to compete. WP 6.1. presents a review of the literature on competitiveness concepts and specifies appropriate indicators. These indicators build on variables like international trade performance or profitability which reflect the outcome of competitiveness in observable variables.

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[1] Leamer, E. (1985), p. 308

2.1. Competitiveness indicators

For the sensitivity analysis in the next sections we focus on the following competitiveness indicators, which are also selected in WP 6.1. for the analysis of the implications of alternative climate policies in the EU.

The *Terms of Trade Indicator (TOT)* for region *i* is defined as:

X denotes export quantities, P^x export prices, P^m import prices, *M* import quantities, *i* the region and *j* the sector. This index compares the ratio of a country's overall exports with the ratio of country's overall imports in all sectors. Terms of trade are said to deteriorate if the indicator decreases; terms of trade are said to improve if the indicator increases. The *ToT* indicator measures the competitiveness of the economy as a whole (*competitiveness at the national level*, cf. WP 6.1) whereas the following indicators measure competitiveness on sectoral level.

The Revealed Comparative Advantage (RCA) index compares the performance of a particular sector with an average performance of all sectors within the same region (*competitiveness at the sectoral level,* cf. WP 6.1). The *RCA* index for region *i* in sector *j* can be presented as follows:

X denotes exports, P^x export prices, P^m import prices, *M* import quantities, *i* the region and *j* the sector. For a particular region and sector, this index compares the ratio of exports by a specific sector to its imports with the ratio of exports to imports across all sectors of the region. If the sectoral export-import ratio is identical to the economy-wide ratio, the *RCA index* takes the neutral value of one ($RCA_{ij}=1$). Thuss, a region *i* is said to have a comparative advantage in sector *j* if the *RCA index* exceeds unity ($1< RCA_{ij} <\infty$) and a comparative disadvantage if the *RCA index* takes the values between zero and one ($1< RCA_{ij} <\infty$).

Relative World Trade Shares (RWS) indicator shows how the relative performance of a particular sector changes compared to the relative performance of the same sectors across the world (*competitiveness at the sectoral level,* cf. WP 6.1). The *RWS* index for region *i* in sector *j* can be written as follows:

X denotes exports, P^x export prices, i the region and j the sector. This index compares the ratio of a country's exports in a certain sector to the world's exports in this sector with the ratio of country's overall exports to the world's exports in all sectors. The *RWS indicator* may be interpreted regarding the range for comparative advantage and disadvantage in a similar way as the *RCA indicator.*

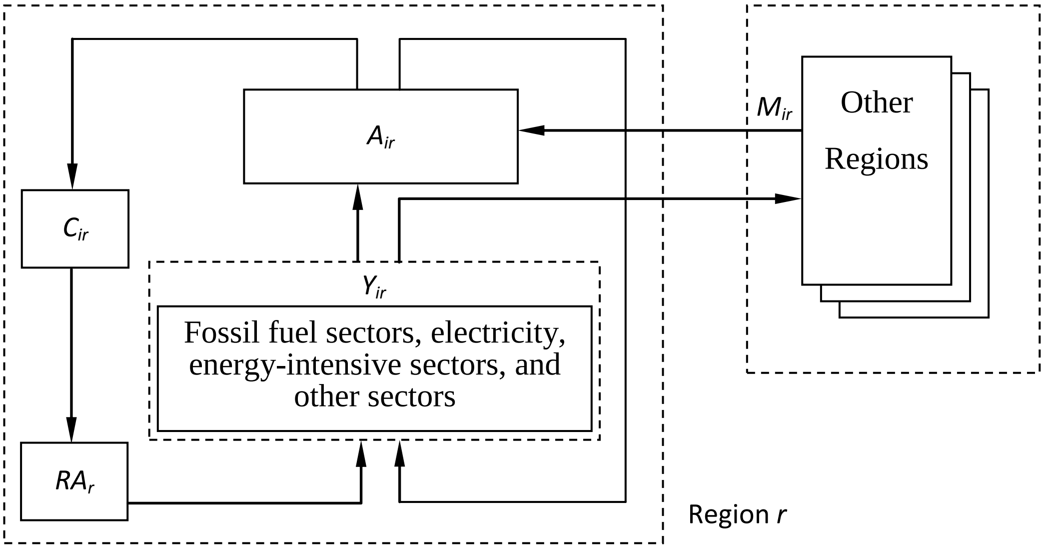
The *Relative Trade Balance (RTB) index* compares the trade balance (exports minus imports) for a product to the total trade (exports plus imports) of that product:

This index has the neutral value of zero ($RTB_{ij}=0$) and the value range of $-1<RTB_{ij} \leq 1$. The region i is said to have a comparative advantage in sector j if the *RTB index* exceeds zero and a comparative disadvantage for values less than zero. The indicator shows how net exports of a particular sector vary relative to the sum of exports and imports in the same sector (*competitiveness at the sectoral level,* cf. WP 6.1).

2.2. Competitiveness Indicators in Climate Policy Analysis

Like in WP 6.1., we implement these competitiveness indicators into a static multi-region, multi-sector model PACE of the word economy with international energy use and global trade. Figure 1 provides a systematic structure of this computational general equilibrium model. A representative agent RA_r in some region r is endowed with primary factors including labor, physical capital and resources of fossil-fuels. Production Y_{ir} of commodity i in region r, other than primary fossil fuels, is captured by aggregate production functions which characterise technology through substitution possibilities between various inputs. Following the Armington (1969) approach, domestic and foreign goods of the same variety are distinguished by origin. All goods from domestic production Y_{ir} either enter the formation of the Armington good A_{ir} or are exported to other regions. A_{ir} is a CES composite of all goods used in the domestic market for intermediate or final demand and combines the domestically produced goods with imports M_{ir} from other regions. C_{ir} denotes the final demand of the representative agent RA_r in each region. It combines consumption of an energy aggregate with a non-energy consumption bundle using nested CES functions for substitution patterns. Endowments of primary resources are fixed exogenously and investment is fixed at the benchmark level. Competitive factor and commodity markets are given in such a way that prices adjust to clear the markets. The model is based on consistent accounts of national production and consumption, as well as trade and energy flows as provided by the GTAP database. For details and an algebraic formulation of the core model see WP 6.1 and Böhringer and Vogt (2003).

Figure 1: Diagrammatic overview of the model structure



Because the focus of the analysis lies on EU policies, the GTAP countries are aggregated to three major regions: European Union (EUR), Non-EU OECD (OEC), and Rest of the World (ROW). Apart from primary and secondary energy production like coal, crude oil, natural gas, refined oil products and electricity the model distinguishes between energy-intensive sectors (EIS) and rest of industry and services (OTH). The energy-intensive sectors are potentially most affected by carbon abatement policies as their installations are covered mostly by the EU Emissions Trading Scheme (EU ETS).

The European Commission committed itself to unilateral greenhouse gas emissions reduction of at least 20% by 2020 compared to the 1990 level. WP 6.1 analyses the implications of alternative EU emissions pricing strategies on competitiveness measured by the competitiveness indicators presented in 2.1. Following the specifications in WP 6.1., unilateral emissions abatement policies will differ in our CGE model in two dimensions: first, the degree of the unilateral reduction target of EU emissions compared to the benchmark situation with no emission abatement policies at all (business–as-usual (BAU) scenario), and second, the emissions prices for carbon-intensive EIS industries compared to the prices that have to be paid by the rest of the economy (OTH). The latter are called “tax ratios” in the following. The emission reduction targets are given by 5%, 10%, 15%, 20%, 25% and 30% from the reference benchmark level. The tax ratio (i.e. the ratio of emissions prices) ranges from unity (i.e. uniform emissions pricing) to full exemption of the EIS industries. In between there are factors of 2, 5, 10, and 20 which indicate the ratio at which the carbon prices in the rest of the economy are higher than for EIS industries. We assume unilateral emissions abatement within the EU while other regions have no comparable carbon emissions regulation.

In WP 6.1. it is shown that EU Terms of Trade (TOT) are unambiguously negatively affected by more stringent emissions reduction targets. The RCA and the RWS indicators report competitiveness losses for energy-intensive sectors (EIS) in the EU given a stringent climate policy, but tax measures can mitigate the effect. The results for the other sectors (OTH) are similar but reversed: More stringent climate policy leads to more competiveness. Even though the signs of these effects for these two indicators are similar, the scales are different. The results of the RTB indicator are also consistent with the results of RCA and RWS on a binary level, but the scales differ considerably as RTB reacts strongly even to relatively moderate policy changes.

In section 4 we will conduct a sensitivity analysis to test the robustness of these results with respect to a variation of the Armington elasticities. The Armington elasticities in the PACE model are elasticities of substitution between import and domestic goods in the formation of the Armington good described above.

3. Sensitivity Analysis with Gauss-Quadrature

As we have explained the introduction, in a sensitivity analysis the variation of outputs in a model analysis is related to the variation of input parameters. In the context of CGE models, we ask whether the choice of fundamental parameters of the model, e.g. elasticities or time preference parameters, lead to stable equilibrium values of economic variables, e.g. GDP or emission levels. Usually, in a comparison we refer to the equilibrium of a benchmark scenario. If economic output variables change only moderately for plausible ranges of fundamental parameters, we can trust the results.

Basically, there are two methodological approaches to sensitivity analysis: a deterministic and a stochastic approach. Deterministic sensitivity analysis assumes that the tuple of basic parameters is an element of a given subset of all possible parameter choices. It seeks to determine upper and lower bounds on the corresponding subset of economic outcomes of the model. Stochastic sensitivity analysis treats the vector of parameters as a stochastic variable with a given distribution, rendering economic equilibria of the model into stochastic variables. It aims at calculating the first moments of these variables, with the variance indicating the robustness of the results. Our approach to sensitivity analysis here is a stochastic one.

The choice a modeller has to make is, however, not only a methodological, but also a numerical one. Sensitivity analysis involve a frequent evaluation of the model for different values of parameters. These are costly in terms of calculation time and usually the modeller faces a trade-off between accuracy and calculation time. However calculation time also depends on the method employed: in the following we give a short overview over the method employed here, developed earlier in Hermeling and Mennel (2008) to reduce the number of model evaluations and thus calculation time.

3.1. Mathematical Preliminaries

An equilibrium of a computable general equilibrium (CGE) model takes the mathematical form of a solution to a system of (non-linear) equations

where is a vector of (equilibrium) state variables of the economy (such as capital or wage) and a vector of parameters of the economy (such as demand elasticity or time preference). is a continously differentiable function

that consists of first order conditions and (budget) constraints. This is in particular formal statement of the CGE model PACE used in the present analysis. In the sequel, we assume that the model is uniquely solvable for the parameter values, a situation fulfilled in the case of our analysis.

Sensitivity analysis is concerned with the effect that (minor) changes of basic parameters have on equilibrium state variables . Hereby, the existence of economic equilibria in a neighbourhood of is the theoretical prerequisite for sensitivity analysis. The implicit function theorem gives a definite answer: For a differentiable and regular function[2] *F*

.

One may ask how to ensure regularity of F. Generally speaking, is regular as long as first order conditions and constraints are independent. While it may be difficult to prove this assertion in some cases, it can be checked without problem numerically.

In the following we will thus study the function

3.2. Stochastic Sensitivity Analysis

The to sensitivity analysis treats the vector of basic parameters as a stochastic variable with values . The distribution of is given. While somewhat counterintiutive in the first place, the approach is in line with econometric estimations. These do not only produce mean values for parameters such as demand elasticities, but confidence intervals and

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[2] The case can be more complicated in the presence of multiple equilibria, cf. Hermeling and Mennel (2008), p. 2.

higher moments for them. Under stochastic sensitivity analysis, becomes a mapping onto a stochastical variable of equilibria.
We then calculate the mean and the variation of the equilibrium vector :

Attaching different weights to different economic variables, the stochastic sensitivity analysis assesses the size of

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where index is running over the dimension of . In words: Given a distribution of basic parameters, we investigate the most likely equilibrium (the mean). We assess its robustness by assessing the relative size of the variance of equilibria with respect to the mean, possibly attaching different weights to different economic variables. Results are robust when the variance is small in comparison to the mean of a variable.

Still, this definition leaves us with the problem of evaluating multidimensional integrals – which can prove to be a hard task. Most modellers use the so-called - approach as a practical implementations of stochastic sensitivity analysis. Both mean and variance of equilibrium , as defined by equation 1 and 2, are approximated in the following way: Draw a (large) set of realisations from the distribution and calculate

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The sums of the right-hand side converge stochastically to the true values of and . Beyond mean and variance of the stochastic variable , we can easily approximate its distribution . A great disadvantage of the Monte-Carlo approach is that in order to assure convergence, the number of draws has to be high and thus the approximation is numerically costly. This is a problem in particular when the space of parameters is high dimensional - the curse of dimensionality drives up the number of necessary draws exponentially (cf. Judd 1998).

3.3. The Gauss-Quadrature Approach

A second way of practically implementing stochastic sensitivity analysis is by - in fact a numerical method to approximate integrals (cf. Stoer 1990). Essentially, the Gauss quadrature gives us nodes and weights to approximate the (one dimensional) integral

In the specific case of stochastic sensitivity analysis, we look for nodes and weights to approximate mean and variance of equilibria[3]:

(*)

where for the ease of presentation, we assume the dimension of economic variables is set to (the extension to multiple dimensions is straightforward).

Gauss-Quadrature (GQ) methods based on the formulae (*) above have been introduced to economics by Arndt 1996 (cf. also DeVuyst and Preckel 1997) who proposes a complex non-linear equation system to solve for nodes and weights.

In contrast the GQ method proposed by Hermeling and Mennel (2008) and used in this deliverable builds on . This approach simplifies the computation of a sensitivity analysis in cases of standard probability distributions and increases the approximation quality at the same time. This is possible because the optimal nodes turn out to be zeros of orthogonal polynomials. They have to be linearly transformed to fit the respective interval but can otherwise be taken from an existing table. Orthogonality in this context is defined by the scalar product

By virtue of the Gram-Schmidt procedure and the Weierstrass theorem, there exists a complete system of orthogonal polynomials on the space of continuous functions. There are well known examples of orthogonal polynomials, the best known being Legendre, Tchebychev, Laguerre and

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[3] Here and in the following, we assume that probability distribution can be represented by a (continuous) probability densitiy function While somewhat limiting the applicability of the procedure, we can safely say that all economicly sensible distributions should fulfill this assumption.

Hermite polynomials.

Examples of families of orthogonal polynomials

Orthogonal polynomials have a striking feature[4]: their zeros are real and distinct. Therefore they are a natural choice of nodes for the evaluation of the approximation formula above. In fact, one can show that for a given density function (i.e. probability distribution), the zeros of the corresponding orthogonal polynomial of degree and the weights calculated as a solution to the system of linear equations below are the basis for our integration formula (*) (cf. Hermeling and Mennel 2008):

Thus, for our purpose of numerical integration, we have to calculate the zeros of orthogonal polynomials and weights corresponding to the probability distribution with weight function While in general the numerical determination of the zeros of orthogonal polynomials for a given distribution may be tricky, in the case of standard probability distributions we have no problem. A look at the table of orthogonal polynomials confirms that for uniform distributions, we can use Legendre polynomials, and Hermite polynomials for normal distributions. This facilitates our task considerably: We can either (easily) calculate the zeros numerically from the defining formulae for Legendre or Hermite polynomials, or take these from published tables (Greenwood and Miller 1948).

In higher dimensions , i.e. for more than one input parameter, integrals can be approximated by product rules, combining one-dimensional nodes and weights. This allows for a joint variation of basic parameters of the model. The appropriate formula for this purpose is

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[4] For this assertion, compare Stoer and Bulirsch 1990, ch. 3.6, and also Hermeling and Mennel (2008).

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In the next section we apply the method to check the robustness of results on competitiveness indicators under climate policy.

4. Results of the Sensitivity Analysis

In this section we conduct a sensitivity analysis for the results of WP6.1, i.e. the competitiveness effects of climate policy measured by a set of economy-wide and sectoral indicators in the CGE model PACE. As explained in section 2, the key basic parameters varied are *Armington elasticities*. As outlined in section 2, import and export varieties are treated as imperfect substitutes in the PACE model (Armington assumption, Armington 1969), their rate of substitutability being governed by the Armington elasticity. Estimates for this elasticity are found to vary considerably in the literature, differing by sector and region (cf. Gallaway et al. 2003, Reinert 1992, McDaniel and Balistreri 2003, Welsch 2008). In the PACE model, the basic choice of the Armington elasticities between domestic and foreign goods is 4. Clearly, a sensitivity analysis is warranted. We use a uniform distribution over the interval [1,10].

The key variables to be analyzed are of course the *competitiveness indicators* that reflect the economic outcome of the competitiveness analysis. More precisely we are interested in whether the effects of climate policy on competitiveness as outlined in WP6.1 are similar for different choices of the basic input parameter, the Armington elasticity. Following the method outlined in section 3, we calculate mean and variance of the indicators by formula (*), i.e. we employ the GQ method for a uniform distribution using zeros and weights of Legendre polynomials. Results can be regarded as robust whenever the variance is small with respect to the mean of a variable under the policy treatment. In our case we vary the overall emission reduction target and the tax rate (as explained in section 2), and then calculate the mean and the variance of the Terms-of-Trade (ToT), the Revealed Comparative Advantage (RCA), the Relative World Trade Share (RWS) and the Relative Trade Balance (RTB) under a uniform distribution of the underlying Armington elasticity.

The results of the sensitivity analysis are displayed in the graphs G1a through G7b as contour plots over the unilateral emissions abatement target and the emissions tax ratio. Thereby, the a. graphs capture the mean and the b. graphs the corresponding variance for different competitiveness indicators. The label “inf” for the tax ratios refers to the case of full emissions price exemptions of EIS industries with an associated infinite price ratio.

As we see in graph G1a, showing the mean of the ToT indicator under different climate policies, the terms of trade are uniformly reduced as the emission reduction target becomes more stringent. This is result is in line with WP6.1, where a similar graph has been produced for the basic parametric choice of the Armington elasticity. G1b shows the corresponding variance, which is much lower than the mean (by an order of three magnitudes), leaving no doubt about the robustness of the result.

As in WP6.1, the RCA indicator of energy intensive sectors (EIS)[5] is reduced by more stringent climate policies, but tax measures mitigate the effect (graph G2a). Graph G2b displays the corresponding variance, it is about to a quarter of the mean. Clearly, this demonstrates that the choice of the Armington elasticity matters much more for the effect of climate policy on the EIS than for the overall trade balance of the economy. Yet, the basic assertion on the effect of climate policy on EIS from WP6.1 is not reversed by the outcome of the sensitivity analysis, but only its magnitude. Graphs G3a and G3b show the mean and variance of the RCA indicator for the other sectors (compare section 2), leading to a similar picture as in the case of EIS: in this case the RCA of OTH is increased with more stringent climate policies, with tax measures reducing the effect. Also, the choice of the Armington elasticity matters for the magnitude, but not the content of this result.

But not only the choice of the basic parameter, but also the choice of the indicator is important for the result: this can be deduced from the graphs G4a through G5b. They show the Relative World Trade Share (RWS) of the different sectors, an alternative sectoral competitiveness indicator. While the basic assertion on the competitiveness effect of climate policy following from the RWS outcomes is the same as for the RCA, the result apparently depends much less on the choice of the Armington elasticity: the variance of RWS both for EIS and OTH is lower than the mean by an order of magnitude, suggesting a high robustness of the results.

As in WP 6.1, the Relative Trade Balance (RTB) for the different sectors reacts very strongly even to relative moderate changes in the basic policy parameters. The graphs G6a and G7a are very similar to the graphs reported in WP 6.1. with possible losses respectively competitiveness improvements of more than 100% for the different sectors compared to BAU. The corresponding variances plotted in graph G6b and G7b, however, are nearly 6 times higher in magnitude than the mean for EIS and nearly 40 times higher than the mean for OTH. It shows that the Relative Trade Balances reacts not only very sensitive on variations of policy parameters but also on variations of other parameters like

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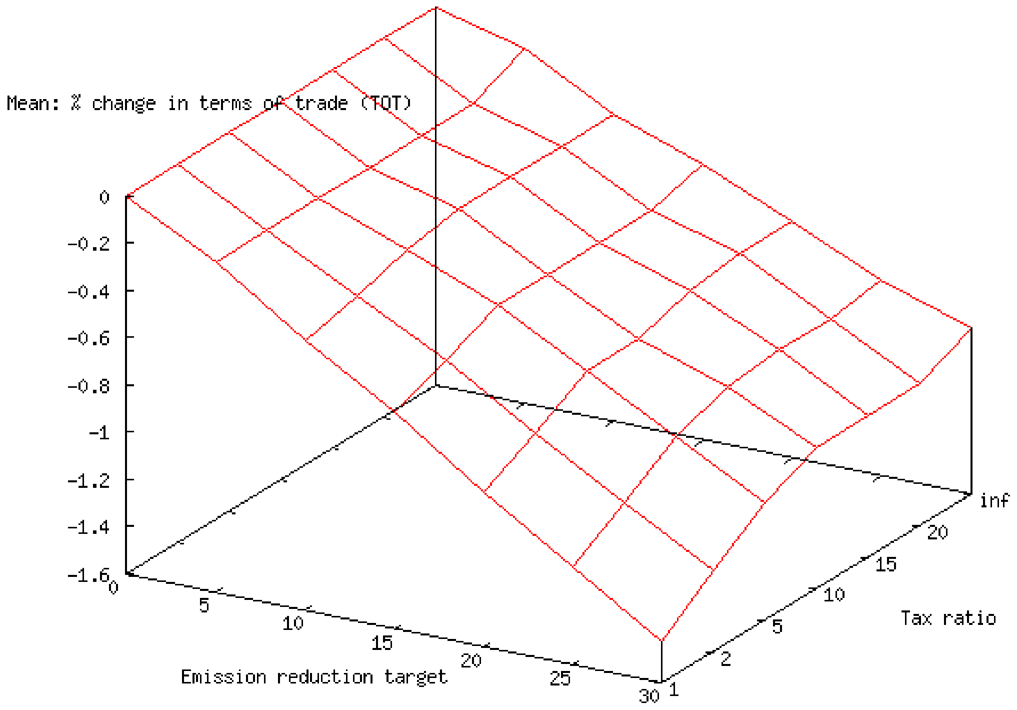
[5] As in WP6.1, we concentrate on the energy intensive and the other sectors, reflected their overall importance for climate policy analysis.

the Armington elasticities. This result shows that the previous results drawn from the RTB indicator are not robust and thus not reliable. We believe, however, that this highlights a problem of the indicator and not of the underlying model, i.e. the indicator is likely to generate unstable results in any computational model due to its conceptual simplicity.

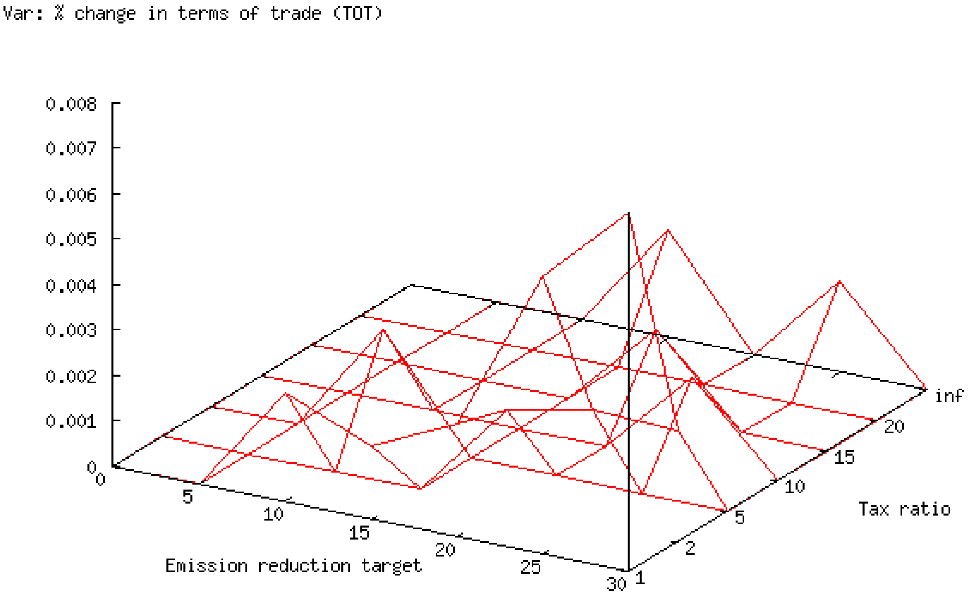
5. Conclusion

We have conducted a stochastic sensitivity analysis for the climate policy analysis of WP6.1, making use of the Gauss-Quadrature approach proposed by Hermeling and Mennel (2008). The results show that basic assertions of the analysis for the economy-wide and sectoral competitiveness are robust with respect to a uniform variation of Armington elasticities. The only exception is given by the Relative Trade Balance (RTB) which reacts very sensitive not only to policy changes but also to small changes of the Armington elasticities. Across the other indicators, there are also important differences: While the economy-wide Terms-of-Term competitiveness indicator is hardly affected by the variation, the sectoral Revealed Comparative Advantage (RCA) indicator changes in magnitude when the Armington elasticity is altered. Yet the basic policy assertions for the EIS and the OTH sectors remain unchanged – i.e. more stringent climate policy targets reduce competitiveness, with tax ratios mitigating the effect. The Relative World Trade Share (RWS) indicator reveals the same qualitative results as the RCA, but these are more stable under variations of the Armington elasticity. This shows that robustness of the results apparently depends on the choice of the indicator. We believe, however, we have shown the robustness of the economic analysis, with the high variation of the RTB highlighting a problem of the indicator and not of the underlying model.

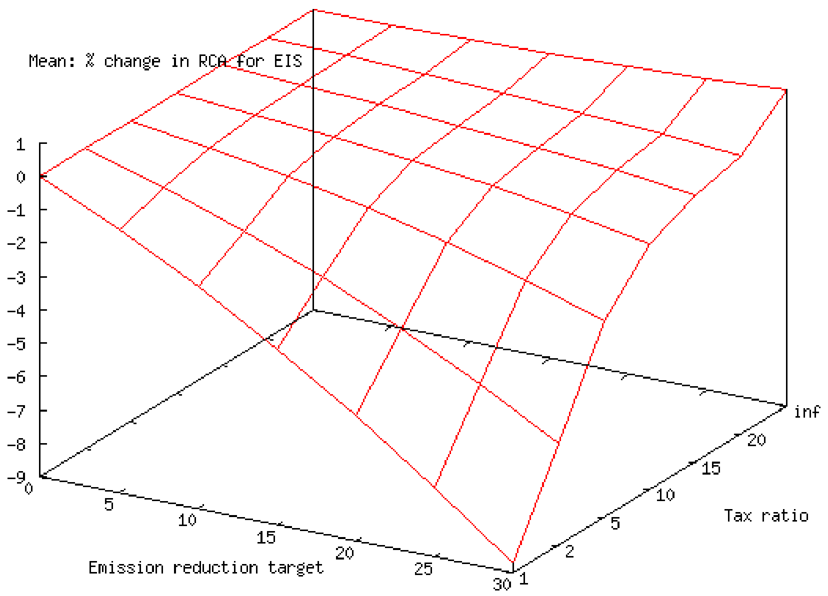
Appendix: Graphs



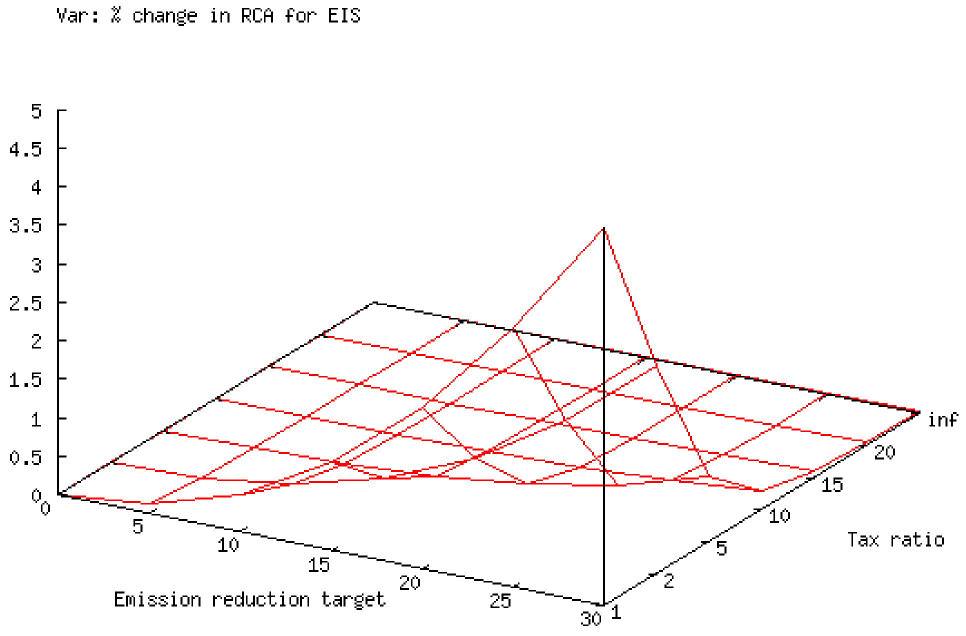
G1a: Mean of Terms-of-Trade (ToT) Changes with respect to Business-as-usual (BaU)



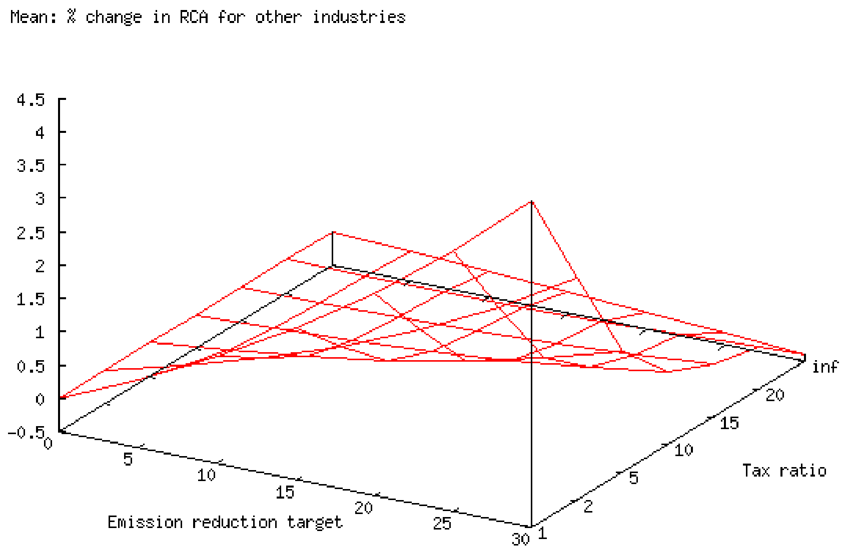
G1b: Variance of Terms-of-Trade (ToT) Changes with respect to Business-as-usual (BaU)



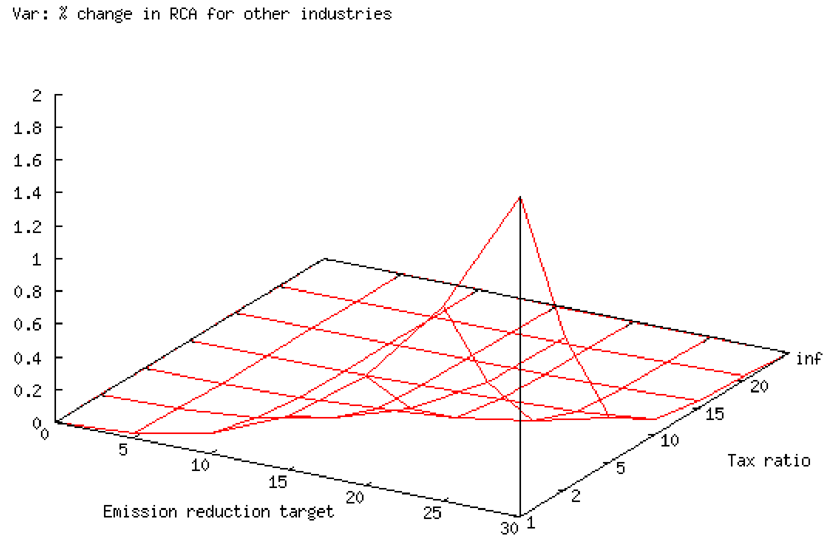
G2a: Mean of Revealed Comparative Advantage (RCA) Changes with respect to Business-as-usual (BaU) for Energy Intensive Industries (EIS)



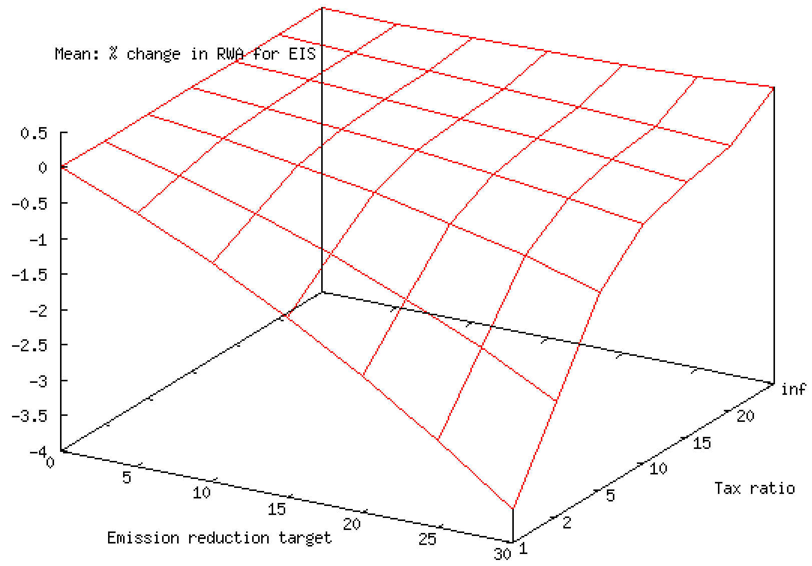
G2b: Variance of Revealed Comparative Advantage (RCA) Changes for Energy Intensive Industries (EIS)



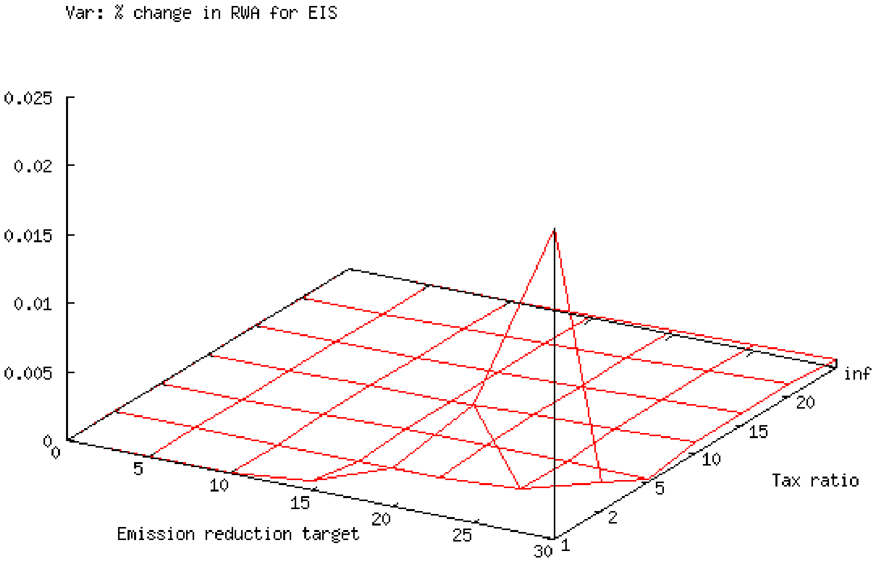
G3a: Mean of Revealed Comparative Advantage (RCA) Changes with respect to Business-as-usual (BaU) for Other Sectors (OTH)



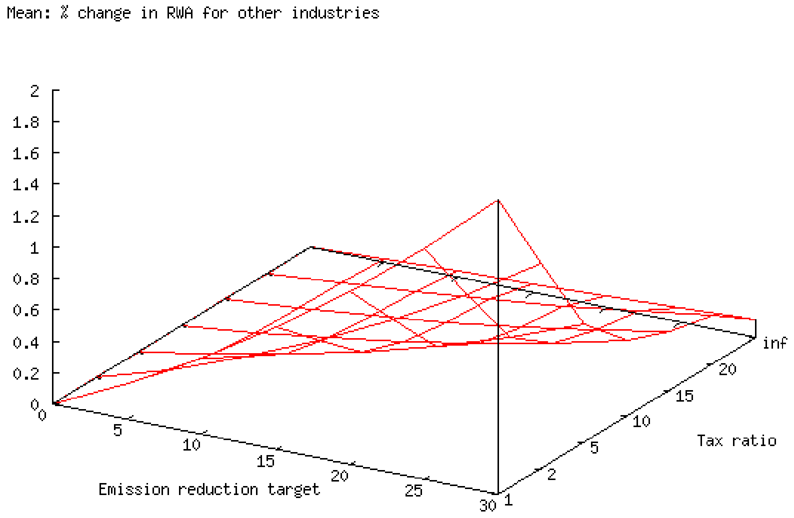
G3b: Variance of Revealed Comparative Advantage (RCA) Changes for Other Sectors (OTH)



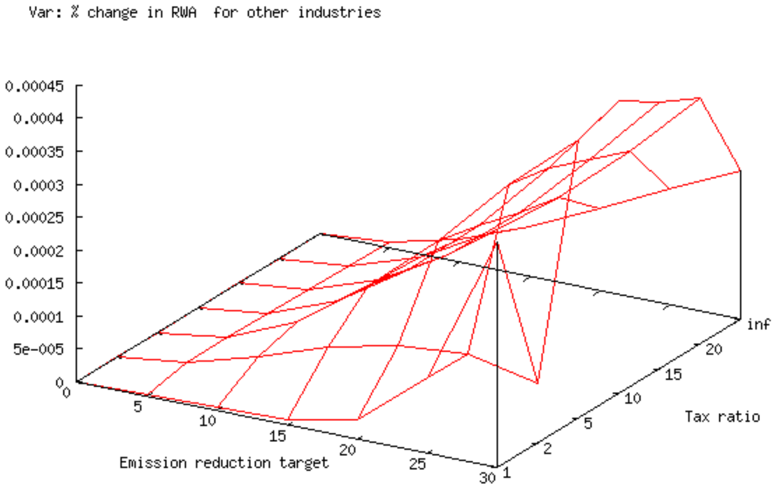
G4a: Mean of Relative World Trade Shares (RWS) Changes with respect to Business-as-usual (BaU) for Energy Intensive Sector (EIS)



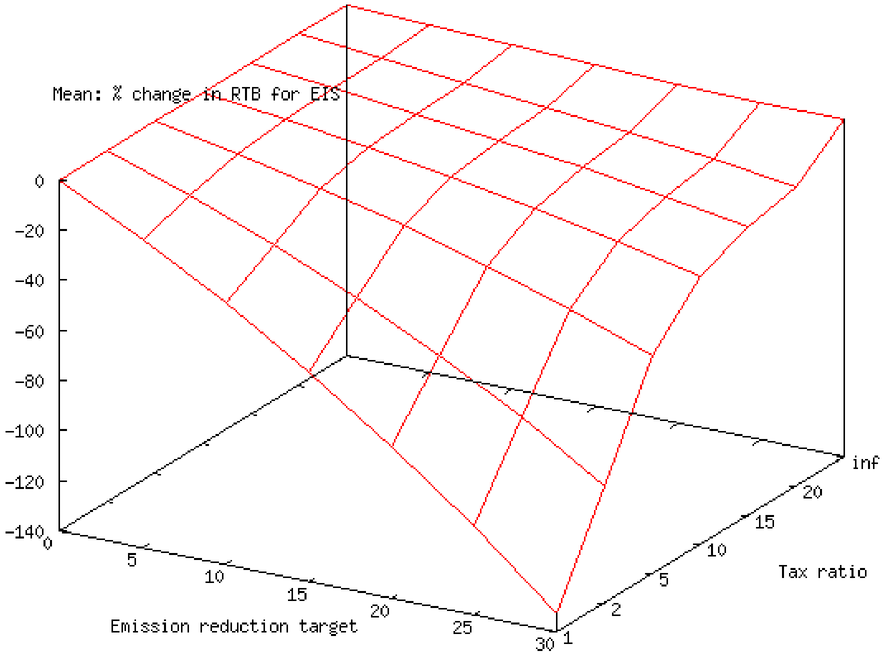
G4b: Variance of Relative World Trade Shares (RWS) Changes for Energy Intensive Sector (EIS)



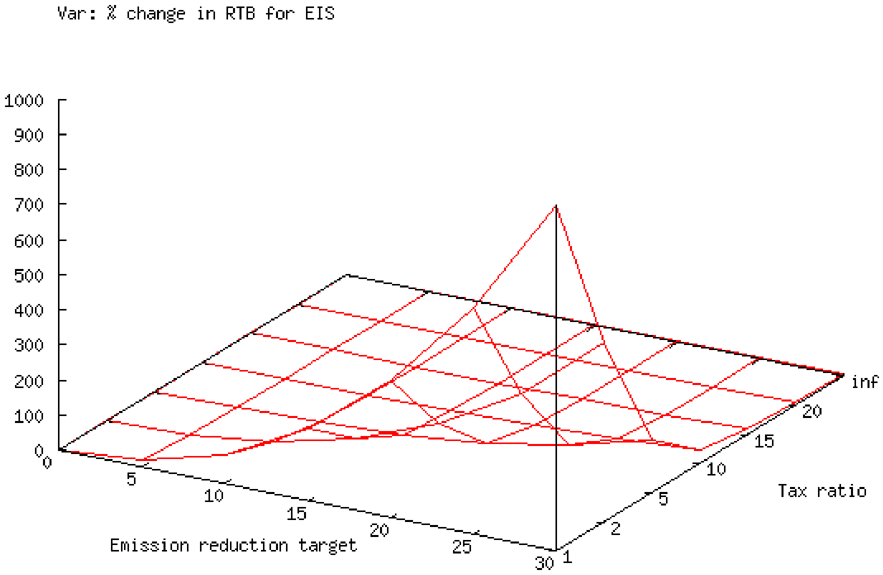
G5a: Mean of Relative World Trade Shares (RWS) Changes with respect to Business-as-usual (BaU) for Other Sectors (OTH)



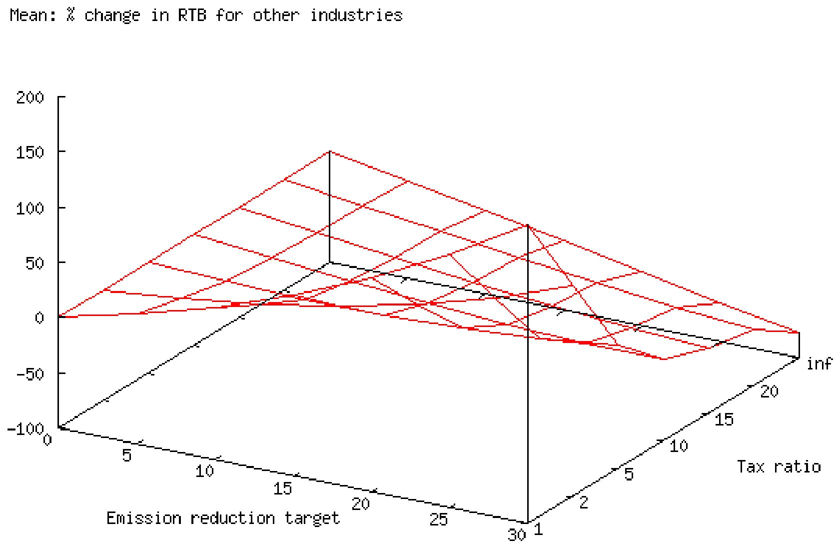
G5b: Variance of Relative World Trade Shares (RWS) Changes for Other Sectors (OTH)



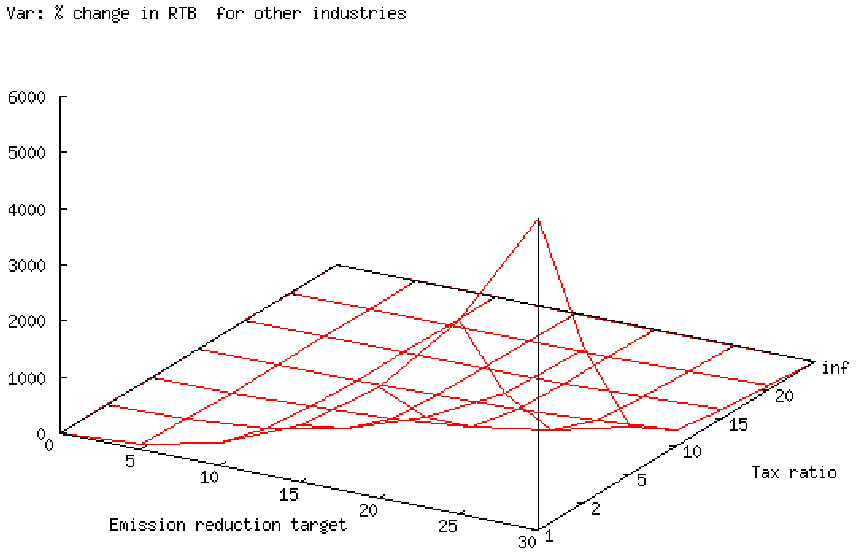
G6a: Mean of Relative Trade Balance (RTB) Changes with respect to Business-as-usual (BaU) for Energy Intensive Sectors (EIS)



G6b: Variance of Relative Trade Balance (RTB) Changes for Energy Intensive Sectors (EIS)



G7a: Mean of Relative Trade Balance (RTB) Changes with respect to Business-as-usual (BaU) for Other Sectors (OTH)



G7b: Variance of Relative Trade Balance (RTB) Changes for Other Sectors (OTH)

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